An Approach to Structure/Control Simultaneous Optimization for Large Flexible Spacecraft

Junjiro Onoda* and Raphael T. Haftka†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia

This paper presents an approach to the simultaneous optimal design of a structure and control system for large flexible spacecrafts based on realistic objective function and constraints. The weight or total cost of structure and control system is minimized subject to constraints on the magnitude of response to a given disturbance involving both rigid-body and elastic modes. A nested optimization technique is developed to solve the combined problem. As an example, simple beam-like spacecraft under a steady-state white-noise disturbance force is investigated and some results of optimization are presented. In the numerical examples, the stiffness distribution, location of controller, and control gains are optimized. Direct feedback control and linear quadratic optimal control laws are used with both inertial and noninertial disturbing force. It is shown that the total cost is sensitive to the overall structural stiffness, so that a simultaneous optimization of the structure and control system is indeed useful.

Nomenclature

$[A]$	= plant matrix, Eq. (4)
a	= cross-sectional area of beam structure
$[B]$	= input matrix for control force, Eq. (5)
b	= a constant relating the bending rigidity with cross-sectional area, Eq. (30)
$[C]$	= damping matrix
$[D]$	= input matrix for disturbance force, Eq. (6)
EI	= bending rigidity
$[F]$	= gain matrix, Eq. (7)
$\{f_d\}$	= disturbance force
$[G]$	= closed-loop system matrix, Eq. (9)
J	= cost function or objective function, Eq. (19)
$[K]$	= stiffness matrix
L	= half length of structure
$[M]$	= mass matrix
m_c	= mass of a controller including its power source
m_N	= normalizing mass
m_s	= mass of half structure
N_d	= number of area design variables in half structure
N_m	= number of normal modes
$[P]$	= solution of the Riccati equation, Eq. (23)
$p(x,t)$	= disturbance force per unit length
$\{q\}$	= state vector, Eq. (3)
$[R_1]$	= weighting matrix of the state, Eq. (10)
$[R_2]$	= weighting matrix of the control force, Eq. (11)
r	= ratio of costs of controller and structure per unit mass
$[S]$	= output matrix
t	= time
$u, \{u\}$	= control force or control force vector
$V, [V]$	= intensity or intensity matrix of white-noise disturbance force per unit length
$\{w\}$	= discretized displacement
x	= coordinate along the structure with the origin at the center of spacecraft
x_c	= location of controller
y	= lateral displacement
α, β	= constants for cost of controller
δ	= delta function
η	= ratio of the damping matrix to the stiffness matrix, Eq. (32)
κ	= weighting constant of σ_2, Eq. (21)
μ	= mass density fraction of structure in the normalizing case
ρ	= linear mass density
ξ_i	= nondimensional cross-sectional area, Eq. (27)
σ_1	= index for displacement
σ_1^*	= maximum allowable value of σ_1
σ_2	= index for control force

Symbols

$(\bar{\ })$	= normalized
$E[\ ]$	= expectation

Subscripts

N	= normalizing value

Received June 6, 1986; revision received Nov. 11, 1986.

*Visiting Professor, Department of Aerospace and Ocean Engineering and Associate Professor, Institute of Space and Astronautical Science, Tokyo, Japan. Member AIAA.

†Professor, Department of Aerospace and Ocean Engineering. Member AIAA.

Introduction

LARGE space structure such as huge antennas or space stations will be very flexible, not only because of the high cost of transportation of massive structures from Earth to space, but also because they will be constructed or deployed in orbit and will not need to withstand large launching loads. Therefore, one of the most important requirements for such structures will be to maintain a desired shape with the necessary accuracy under given disturbances. Some of the expected dynamic disturbances can easily excite the vibration of these flexible structures, which are expected to have very low natural damping. Therefore, vibration suppression will be a difficult and important problem; one of the most attractive solutions to this problem is active vibration control.

Traditionally, the structure and the attitude control systems of spacecraft have been designed separately. The structure is typically optimized to minimize weight subject to stress and stiffness constraints and the control system is optimized to minimize a quadratic performance measure accounting for deformation and control effort. However, because of the strong interaction between the structure and control system in active vibration control, simultaneous optimal design of both systems may be necessary in order to obtain maximum performance with minimum cost.

Recently, there has been considerable interest in simultaneous structure/control optimization. Some structural analysts (see, e.g., Refs. 1 and 2) attempted to enhance the performance of the control system by changing the stiffness properties of the structure. The control system and the structure were still designed separately, but the structural optimization was governed by control-related stiffness requirements. Similarly, it is possible[3] to use the design of the control system to infer the required structural changes because some of the gain matrices obtained in the control design are equivalent to modifications in the structural stiffness matrices.

More truly integrated structure/control design procedures were proposed that operate simultaneously on a set of structural and control design variables. The mass of the structure was combined with the quadratic performance index used in optimal control to obtain a composite objective function[4,5] or the quadratic performance index was optimized for a given structural mass.[6] Alternatively, the structure and control system were optimized with constaints placed on the eigenvalues of the closed-loop system.[7,8]

The various simultaneous structure/control optimization formulations represent attempts to reconcile the traditional approaches of control analysts and structural analysts with respect to the choice of objective function and constraints. However, some of these formulations do not represent well the objective and constraints of the actual design problem. In particular, combining mass, control effort, and response into a single objective function is not good because in real applications the response is constrained by practical considerations. Furthermore, the constraints in terms of closed-loop eigenvalues are indirect.

In the case of space stations, for example, it will be required to keep pointing-angle errors, angular velocity, and/or acceleration of specific sections (such as μ-gravity experiment sections and the space observatory) to less than certain values. In the case of huge space antennas, the essential requirement is to maintain shape and attitude with a given accuracy.

The objective function to be minimized by a simultaneous optimal design of the structure and the control system should be the total cost. It is often difficult to estimate the exact costs of the structure and the controller. However, it is reasonable to assume that the cost of the structure is proportional to its mass because of the large cost for transporation to the orbit. The cost of the control system, including its power source, is assumed to be a function of the magnitude of control force required for the actuators.

The present paper formulates the combined structures/control optimization for minimizing the total cost based on the above assumptions, with constraints placed on the magnitude of the response. A nested optimization technique is developed for the solution of the combined problem. Numerical examples, where the stiffness distribution of a beam-like structure, the location of the controller, and the control gains are optimized, are used for demonstration. The investigation is limited to time-invariant linear feedback control law and to steady random disturbances, although transient problems can be treated in the same manner.

Basic Equations

The discretized equation of motion of a structure subjected to the control force and disturbance is

$$[M]\{\ddot{w}\}+[C]\{\dot{w}\}+[K]\{w\}=[B_s]\{u\}+[D_s]\{f_d\} \quad (1)$$

where $\{w\}$ is the discretized displacement, $\{u\}$ the control force, and $\{f_d\}$ the disturbance. In state vector form, Eq. (1) becomes

$$\{\dot{q}\}=[A]\{q\}+[B]\{u\}+[D]\{f_d\} \quad (2)$$

where

$$\{q\}=[\{\dot{w}\}^T,\ \ \{w\}^T]^T \quad (3)$$

$$[A]=\begin{bmatrix} -[M]^{-1}[C], & -[M]^{-1}[K] \\ [I] & [0] \end{bmatrix} \quad (4)$$

$$[B]=[([M]^{-1}[B_s])^T,\ \ [0]]^T \quad (5)$$

$$[D]=[([M]^{-1}[D_s])^T,\ \ [0]]^T \quad (6)$$

In the case of linear feedback control, the control force is a linear function of the state vector, i.e.,

$$\{u\}=-[F]\{q\} \quad (7)$$

and from Eq. (2)

$$\{\dot{q}\}=[G]\{q\}+[D]\{f_d\} \quad (8)$$

where

$$[G]=([A]-[B][F]) \quad (9)$$

As a measure of the magnitude of the displacements and velocities of the structure we use

$$\sigma_1=E[\{q\}^T[R_1]\{q\}] \quad (10)$$

where a positive semidefinite weighting matrix $[R_1]$ is chosen according to the relative importance of the displacements of the various parts of the structure and E denotes the expected value. Similarly, the control effort is measured by

$$\sigma_2=E[\{u\}^T[R_2]\{u\}] \quad (11)$$

where $[R_2]$ is a positive definite weighting matrix.

For a steady white-noise disturbance the intensity matrix $[V]$ is defined as

$$E[\{f_d(t)\}\{f_d(t-t_0)\}^T]=\delta(t_0)[V] \quad (12)$$

Then σ_1 and σ_2 can be estimated if the system is asymptotically stable,[9] as

$$\sigma_1=\mathrm{tr}[[P_1][D][V][D]^T] \quad (13)$$

$$\sigma_2=\mathrm{tr}[[P_2][D][V][D]^T] \quad (14)$$

where $[P_1]$ and $[P_2]$ are the solutions of the following Lyapunov equations, respectively:

$$[P_1][G]+[G]^T[P_1]+[R_1]=0 \quad (15)$$

$$[P_2][G]+[G]^T[P_2]+[F]^T[R_2][F]=0 \quad (16)$$

and tr denotes the trace operator.

If we measure the magnitude of displacements and velocities by

$$\sigma_1'=\int_0^\infty \{q\}^T[R_1]\{q\}\mathrm{d}t \quad (17)$$

and the control effort by

$$\sigma_2'=\int_0^\infty \{u\}[R_2]\{u\}\mathrm{d}t \quad (18)$$

in the case of transient problem without any disturbance forces, these indices can also be estimated by Eqs. (13)

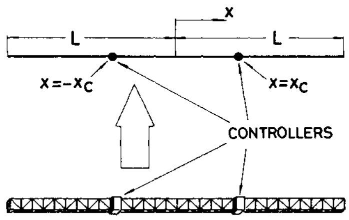


Fig. 1 Beam-like spacecraft.

and (14) by replacing $[D][V][D]^T$ in the equations by $\{q_0\}\{q_0\}^T$ or $E[\{q_0\}\{q_0\}^T]$, where $\{q_0\}$ denotes the determinate or stochastic initial state vector at $t=0$, respectively. Therefore, this type of transient problem can be treated in the same way as the steady-state problem. Subsequent discussions concern the steady-state problem.

Optimization

The cost of the structure and the control system are assumed to be linear functions of their mass and the mass of controller including its power source is assumed to be a linear function of σ_2^β. Therefore, the following cost function (i.e., objective function) is used:

$$J=(m_s+\alpha\sigma_2^\beta)/m_N \tag{19}$$

where m_s and m_N are the structural mass and a normalizing mass respectively, and the values of α and β should be determined to reflect the cost of the control system. Since the magnitude of displacements can be represented by σ_1, the optimization problem is to minimize J under the constraint

$$\sigma_1 \le \sigma_1^* \tag{20}$$

where σ_1^* denotes the maximum allowable value of σ_1.

Optimization is carried out in two different ways. In the first method, all the structural parameters and the elements of the gain matrix $[F]$ are equally treated as design variables and the values that minimize Eq. (19) under the constraint of Eq. (20) are searched numerically. In the present paper, this approach is applied only to the cases of direct feedback control, where the number of gain variables is relatively small.

The second method is a nested optimization method designed to take advantage of optimal linear quadratic (LQ) control theory. It is known that the optimal linear control gain matrix $[F]$, which minimizes

$$\sigma_0=\sigma_1+\kappa\sigma_2=E[\{q\}^T[R_1]\{q\}+\kappa\{u\}^T[R_2]\{u\}] \tag{21}$$

can be obtained as follows:

$$[F]=\kappa^{-1}[R_2]^{-1}[B]^T[P] \tag{22}$$

where $[P]$ is the solution of the following algebraic Riccati equation[9]:

$$\kappa^{-1}[P][B][R_2]^{-1}[B]^T[P]-[A]^T[P]$$
$$-[P][A]-[R_1]=0 \tag{23}$$

This suggests the following nested optimization formulation. The inner optimization consists of finding the optimal control system for a given structure and κ value by using Eqs. (22) and (23). The outer optimization consists of finding the optimal structural parameters and κ value to minimize J subject to the constraint on σ_1. This nested approach relies on σ_1 decreasing monotonically as κ is decreased. Under that condition, the nested optimization has the same optimum as the original problem. Its major advantage is that the control system can be designed by solving the Riccati equation. This nested optimization method is attractive especially when the matrix $[F]$ has a relatively large number of degrees of freedom. In the present paper, this method is applied to all the cases of optimal linear control.

Beam-Like Spacecraft Example

As an example, a free-free beam-like structure of length 2L as shown in Fig. 1 is investigated, being approximated by a beam. A uniformly distributed payload is supported by the structure. Two attitude control units with torque actuators are symmetrically installed on the structure. The structure is subjected to an antisymmetric disturbance force. Since the dynamic system is symmetric and the disturbance is antisymmetric, only the antisymmetric response of one-half of the structure needs to be investigated.

In this investigation, two types of disturbance forces, i.e., a noninertial force and an inertial force are considered. In the case of noninertial disturbance, the disturbance force per unit length $p(x,t)$ is assumed to be

$$p(x,t)=\sqrt{3}(x/L)f_w(t) \tag{24}$$

where $f_w(t)$ is white noise with intensity V. This type of disturbance can be caused, for example, by solar wind, drag, or some motion inside the spacecraft. For the sake of simplicity, the noninertial disturbance force is assumed proportional to the distance from the center of the spacecraft. For the inertial disturbance, it is assumed that

$$p(x,t)=\sqrt{3}(x/L)\rho(x)f_w(t)/\rho_N \tag{24}$$

where $\rho(x)$ is the total mass per unit length and ρ_N a normalizing density,

$$\rho_N=m_N/L \tag{25}$$

The inertial disturbance can be caused, for example, by gravity gradients or point action of the entire spacecraft toward a target moving with random acceleration.

The controllers and their power sources are located at $x=\pm x_c$. The mass of the control system and power source m_c is assumed proportional to σ_2^β,

$$m_c=m_N\alpha\sigma_2^\beta/r \tag{26}$$

where r is the ratio of the cost per unit mass of the structure and the controller.

The total cross-sectional area of the structure $a(x)$ is approximated by a piecewise-constant function over constant intervals. The nondimensional area in the ith interval ξ_i is

$$\xi_i=a(x)/a_N=a(-x)/a_N, \quad (i-1)/N_d<x/L<i/N_d \tag{27}$$

where N_d is the number of intervals and a_N the normalizing cross-sectional area. Furthermore, it is also assumed that the volume density of the structure is $\mu\rho_N/a_N$ and the payload mass per unit length is $(1-\mu)\rho_N$. Therefore, the total mass per unit length and the structural mass of one-half of the spacecraft are given in the term of ξ_i as

$$\rho(x)=\rho(-x)=\rho_N(1-\mu+\mu\xi_i)$$
$$+\delta(x-x_c)m_c, \quad (i-1)/N_d<x/L<i/N_d \tag{28}$$

$$m_s=m_N\frac{\mu}{N_d}\sum_{i=1}^{N_d}\xi_i \tag{29}$$

The bending stiffness of the beam-like structure of the spacecraft $EI(x)$ is related to the cross-sectional area of

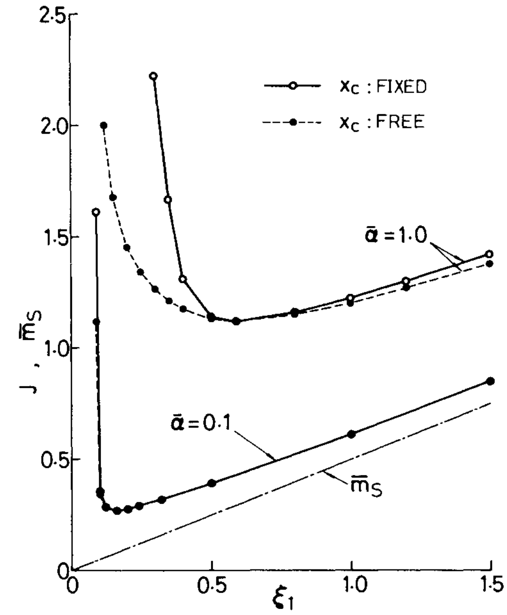
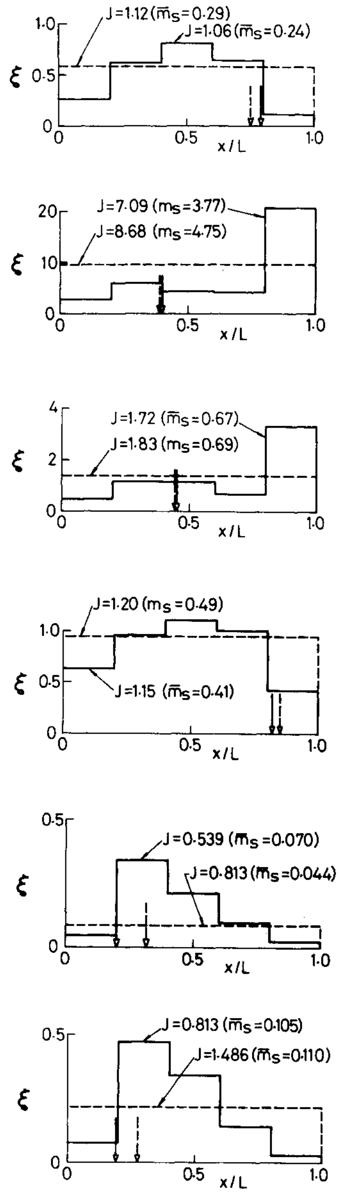


Fig. 2 Examples of optimal total costs for given uniform structures (the structure has only one variable).



a) Noninertial disturbance, optimal LQ control.

b) Noninertial disturbance, optimal LQ control, $\bar{\sigma}_1^* = 10^{-5}$.

c) Noninertial disturbance, direct feedback control.

d) Noninertial disturbance, optimal LQ control, $b = 3$.

e) Inertial disturbance, optimal LQ control.

f) Inertial disturbance plus concentrated disturbance force, optimal LQ control.

Fig. 3 Some results of optimization.

structure by

$$EI(x) = EI_N (a(x)/a_N)^b \tag{30}$$

So that

$$EI(x) = EI(-x) = EI_N \xi_i^b, \qquad (i-1)/N_d < x/L < i/N_d \tag{31}$$

where EI_N is the bending stiffness of the structure with cross-sectional area a_N.

A standard cubic beam finite element with a consistent mass matrix is used to discretize the system. The half of the spacecraft is modeled by five elements. The damping matrix $[C]$ is assumed to be proportional to $[K]$ as

$$[C] = \eta [K] \tag{32}$$

The mathematical model is further translated into normal mode coordinates in order to decrease the degree of freedom and in order to decouple the controlled modes from the residual uncontrolled modes. The modal parameters are recalculated as the strucure is modified during the optimization process. It is assumed that the response of the structure can be sufficiently represented by the lowest N_m modes.

In the case of direct feedback control, the control moment is assumed to be a sum of a term proportional to the instantaneous value of the output of angular velocity sensor and another term proportional to the output of angle sensor, as

$$u = -[F]\{q\} = [-f_1\{S\}^T, \quad -f_2\{S\}^T]\{q\} \tag{33}$$

where the element s_i of vector $\{S\}$ is the rotational angle of the senor caused by the unit modal displacement of the ith mode. The constants f_1 and f_2 are control gains whose values are to be determined so that optimal control performance can be obtained. In this investigation, both the angular velocity and the angle sensors are assumed to be colocated with the actuator. Such a system is not only simple, but also guarantees stability if the polarity is right and there are no time delays. In the case of linear optimal control, the states of the lowest N_m modes (i.e., $\{q\}$) are assumed to be exactly measurable and the optimal values of the gain matrix $[F]$ are obtained from Eq. (22).

In the following investigation, it is assumed that only the shape of the structure is controlled and that velocities do not affect the performance of the spacecraft. This assumption is appropriate for shape control of large space antennas. The matrix $[R_1]$, which defines σ_1 in Eq. (10), is selected so that σ_1 is the average of the square of the lateral displacement y,

$$\{q\}^T [R_1] \{q\} = \left(\frac{1}{L}\right) \int_0^L y^2(x, \{q\}) \mathrm{d}x \tag{34}$$

The value of $[R_2]$ is set to unity in the present example.

The optimization design variables are $\xi_i (i = 1, N_d)$, x_c, κ, and m_c in the case of optimal linear control [with the optimal value of $[F]$ obtained from Eqs. (22) and (23)]. In the case of direct feedback control, f_1 and f_2 are also design variables instead of κ. The constraints are Eqs. (20) and (26). The following parameter values are used in the numerical calculations except for the values indicated in the next section;

$$\beta = 1, \ r = 1, \ \mu = 0.5, \ N_m = 5,$$

$$\bar{\eta} = 5 \times 10^{-4}, \ b = 1, \ \bar{\sigma}_1^* = 10^{-2}, \ \bar{\alpha} = 1$$

where

$$\bar{\eta} = \eta E I_N^{1/2} / (\rho_N^{1/2} L^2) \tag{35}$$

$$\bar{\sigma}_1^* = \sigma_1^* E I_N^{3/2} \rho_N^{1/2} / (VL^6) \tag{36}$$

$$\bar{\alpha} = \alpha (VL^2 E I_N^{1/2} / \rho_N^{1/2})^{\beta} / (L\rho_N) \tag{37}$$

These values roughly correspond to a 200 m long aluminum truss structure that is ¾ m in depth and 1.5 cm^2 in total cross-sectional area (two-thirds of which is that of longerons), controllers whose mass per unit control effort is 10 $kg/(N\cdot m)^2$ including the power source, and a requirement that the mean square of displacement should be less than (3 mm)^2 under a disturbance force of $(\sqrt{3}\times 10^{-3}\ N/m)^2/(rad/s)$ at the tips of the structure.

The NEWSUMT-A optimization program[10] has been used to solve the optimization problem numerically. NEWSUMT-A employs an extended interior penalty function formulation using Newton's method with approximate second derivatives for each unconstained minimization. The derivatives of the constraints were calculated by finite differences.

Results

First, a beam of uniform cross section was optimized with the width of the beam (or the cross-sectional area of the truss members with constant length), the position of the controller and the gains of the control system being the design variables. The effect of the gross stiffness of the structure on the total cost was investigated by varying the structural cross system for the new cross-sectional area. Figure 2 shows how the optimal section from its optimal value and reoptimizing the control value of J varies with the cross-sectional area. The solid lines show the results of optimization with respect to the control gains with the location of the controller fixed (at its optimal location) and the broken lines indicate the results of optimization with respect to both the control gains and the location of controller. The dotted lines shows the normalized structural mass $\bar{m}_s = m_s/m_N$.The figure indicates that when the cross-sectional area of the structure increases from its optimal value the total cost increases gradually because the increase in structural mass is not fully matched by decrease in control effort. However, when the cross-sectional area is decreased from its optimal value, the total cost can become very high because of excessive control effort. The effect of optimization of the location of controller is negligible in most cases of $\bar{\alpha}=0.1$. In the case of $\bar{\alpha}=1.0$, the optimization of the control location decreases the penalty for a small cross-sectional area; but, as the cross section is reduced further, the total cost is very high. The figure indicates the importance of optimizing both the structure and control system. The figure also shows that the optimal uniform structure and the resulting value of J vary substantially with $\bar{\alpha}$.

Next, the effect of varying the stiffness of the structure along its axis was investigated and Fig. 3 shows some results of optimization. The solid lines in the figures show the resulting optimal cross-sectional area distribution and the arrows with solid lines indicate the corresponding optimal location of the controller. The broken lines show the results of optimization where the structure is assumed to be uniform and the arrows with broken lines indicate the location of the controller for the uniform structure. The resulting value of J and $\bar{m}_s = m_s/m_N$, as well as the values of parameters, types of disturbance, and the control law are also shown. Figure 3a is a nominal case corresponding to the parameter values listed in the previous section.

Figure 3b shows a case with relatively small value of $\bar{\sigma}_1^*$ which corresponds to decreasing the mean square of the displacement by 1/1000 from the nominal case. The thousand times stricter requirement resulted in a factor of seven to eight increase in the value of J. The resulting optimal area distributions are entirely different form each other.

Figure 3c shows the same case as the nominal except that direct feedback control was applied. The values of J increase by more than 60% over Fig. 3a where optimal LQ control law is applied under the assumption that all states are exactly measurable.

Figure 3d shows the same case as the nominal except that $b=3$, i.e., the stiffness is assumed to be proportional to a^3. The resulting optimal area distribution and the location of the controller is similar to Fig. 3a. The value of J is slightly larger than the case of $b=1$ because more cross-sectional area is required in order to obtain the same stiffness as in the nominal case of $b=1$.

Figures 3a–3d show that two types of optimal area distribution were obtained for the noninertial disturbance; one with a small tip area and one with a large tip area. The large-tip-area design is counter intuitive because the bending moment in the tip section is low. However, the large tip mass can provide inertia relief to the disturbance force (which is largest at the tip). In the case of $\bar{\sigma}_1^*=10^{-3}$ and $\bar{\alpha}=1$ (not shown here), for example, two local optima with different type of area distribution were obtained depending on the initial design values.

Figure 3e shows a case of inertial disturbance. In this case, relatively low values of J were obtained, with extremely low values of the normalized structural weight $\bar{m}_s$. The reason is that the disturbance force is proportional to the mass and, therefore, the area reduction is efficient in minimizing the value of J.

The inertial disturbance force assumed in Fig. 3e is exactly orthogonal with all the elastic modes. To check whether this orthogonality is responsible for the low structural mass, an uncorrelated concentrated white-noise force of intensity 0.1 VL^2 was applied at the position of $x/L=0.4$ in addition to the inertial disturbance. The results are shown in Fig. 3f and indicate that the optimal configuration was not changed drastically by the concentrated noninertial force, although the value of J was increased substantially.

The comparisons of the values of J of the tailored structure and uniform one in each figure indicate the amounts improvement by the area distribution tailoring. The improvement is only 4–6% in the cases shown in Figs. 3a, 3c, and 3d. However, it is 18% in the case of more strict requirement of $\bar{\sigma}_1^*=10^{-5}$ (Fig. 3b). Further larger improvements are shown in the cases of inertial disturbance, which are 34 and 45% in the cases of Figs. 3e and 3f, respectively.

Conclusions

A new formulation reflecting realistic requirements was introduced for the simultaneous optimal design of a structure and the control system of flexible spacecraft. The combined cost of structure and control system is minimized under the constraint on the magnitude of the response of the structure to a given disturbance involving both the rigid-body and elastic modes. A nested optimization technique which takes advantage of linear control theory is developed to solve the optimization problem. As an example, a simple beam-like spacecraft under a white-noise disturbance force was investigated and some results of the optimization were presented.

The numerical results showed that a nonoptimal structure can result in a very high cost even if the control system is optimized and, therefore, it is important to optimize the structure and control system together. The amount of further improvement by varying the cross-sectional area distribution was shown to depend on the situation. It was 4–18% in the cases of noninertial disturbance and 34–45% in the cases of inertial disturbance investigated here. It was also shown that there are two types of optimal cross-sectional area distribution in the cases of noninertial disturbance. The introduction of optimal LQ control law decreased the total cost by nearly

40% compared with the colocated direct feedback control law.

Acknowledgment

This work was supported in part by NASA Grant NAG-1-224.

References

[1]Khot, N. S., Venkayya, V. B, and Eastep, F. E., "Structural Modifications to Reduce the Los-Error in Large Space Structures," AIAA Paper 84-0997, May 1984.

[2]Venkayya, V. B. and Tishler, V. A., "Frequency Control and Its Effect on the Dynamic Reponse of Flexible Structures," *AIAA Journal*, Vol. 22, Sept. 1984, pp. 1293-1298.

[3]Hanks, B. R. and Skelton, R. E., "Designing Structures for Reduced Response by Modern Control Theory," AIAA Paper 83-0815, May 1983.

[4]Salama, M., Hamidi, M., and Demsetz, L., "Optimization of Controlled Structures," *Proceedings of the JPL Workshop on Identification and Control of Flexible Space Structures*, Jet Propulsion Laboratory, Pasadena, CA, Vol. II, April 1985, pp. 311-327.

[5]Hale, A. L., Lisowski, R. J., and Dahl, W. E., "Optimal Simultaneous Structural and Control Design of Maneuvering Flexible Spacecraft," *Journal of Guidance, Control and Dynamics*, Vol. 8, Jan.-Feb. 1985, pp. 86-93.

[6]Messac, A., Turner, J., and Soosaar, K., "An Integrated Control and Minimum Mass Structural Optimization Algorithm for Large Space Structures," *Proceedings of JPL Workshop on Identification and Control of Flexible Space Structures*, Jet Propulsion Laboratory, Pasadena, CA, JPL Publication 85-29, Vol. II, April 1985, pp. 231-266.

[7]Bodden, D. S. and Junkins, J. L., "Eigenvalue Optimization Algorithms for Structure/Controller Design Iterations," *Journal of Guidance, Control and Dynamics*, Vol. 8, Nov.-Dec. 1985, pp. 697-706.

[8]Haftka, R. T., Martinovic, Z. N., and Hallauer, W. L. Jr., "Enhanced Vibration Controllability by Minor Structural Modifications," *AIAA Journal*, Vol. 23, Aug. 1985, pp. 1260-1266.

[9]Kwakernaak, H. and Sivan, R., *Linear Optimal Control System*, Wiley-Interscience, New York, 1972.

[10]Thareja, R. and Haftka, R. T., "NEWSUMT-A—A General Purpose Program for Constrained Optimization Using Constraint Approximations," *ASME Journal of Mechanics, Transmission, and Automation in Design*, March 1985, pp. 94-99.